DISCOVERY METHOD TEACHING: A CASE STUDY USING GRAPH THEORY

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Currently many teachers are seeking alternatives to traditional lecture methods of presenting subjects in a classroom. The following describes an attempt to encourage students to learn some mathematics actively and independently.

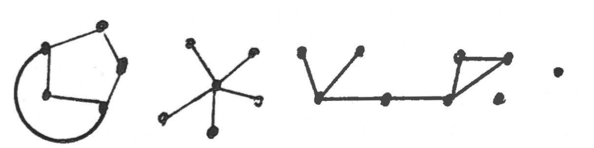
Since Spring, 1970, I have been teaching a discovery method mathematics class, first at Towson State College, near Baltimore, Maryland, and since 1975, at Humboldt State University.

The title of the course in question was originally *Methods of Mathematical Research in Graph Theory,* now shortened to *Graph Theory.* Initially the only formal prerequisite for the course was permission of the instructor. At present I require Abstract Algebra or my permission. I usually interview each student prior to the first class meeting and explain that the course is designed partially as an experience in learning, oriented toward discovery methods; that the students will be working without a textbook or lectures; that I will provide relatively little guidance and expect them to do quite a bit of independent work; that the word 'graph' is used in two different ways in mathematics, and that this course has nothing to do with *graphing* as they are familiar with the term. All those who still express an interest in the class after this vague explanation of what to expect are permitted to enroll. Class enrollment is usually 15 or fewer students. Some of the students are math majors; some teach secondary school mathematics; others are science or engineering majors. Once an English major (math minor) took the class, and one psychology graduate student has taken it.

The class meets once a week for three hours, with a short break halfway through that period. We sit around a large table or move desks into a circle. At the first class meeting I explain that "graph" has two different mathematical meanings, and that the graphs with coordinates and equations are not those meant by Graph Theory. I then give out dittoed sheets with various representations of graphs labeled in three sections: "Each of these is a graph"; "None of these are graphs"; and which of these are graphs"? Once they can identify the graphs correctly, I ask the students to find a definition that would include and exclude exactly the proper examples.

A sample of one such representation is shown below:

EACH OF THESE IS A GRAPH



NONE OF THESE IS A GRAPH

It was only after the second time that I taught this class that I realized how much easier it is for us to discover what something *is* when we can also see examples of what it is *not.*

The beginning examples are chosen to provide some graphs with which students can test conjectures they may make later in the course. With some effort the students can establish a definition similar to the standard one: a *graph* is a set of objects called *points* and a set of objects called *lines* such that

(1) two distinct points are associated with each line, called its endpoints,
(2) no two lines share more than one endpoint.

The next problem is to determine what it should mean for two graphs to be essentially the same, i.e., isomorphic. It often takes until the second class period for the students to reach agreement on this definition. I suspect it takes this long partly because the students are not yet at ease in a class where they have to do most of the talking, with the teacher occasionally asking questions but not saying whether the answers are right or wrong. A second major problem is that the students are not familiar with the concept of isomorphism in many other contexts. A third is that there is no known easy test for isomorphism.

This step of discovering a definition of a concept in an inductive fashion, rather than the more usual academic procedure of stating a definition and having students deduce consequences or applications, would be useful in many disciplines. For example, students might be encouraged to discover elements of poetry like various types of meters or forms of poems by looking at examples and non-examples of poems having those properties.

The next stage of the class begins by my suggesting that the students are now in the position of a mathematician considering a new abstract situation. They have definitions of graphs and isomorphic graphs, and what I expect they will be able to do is find properties that apply perhaps to all graphs, perhaps to a set of graphs which they could define and characterize in some fashion. They are free to use any representation of graphs they prefer. I ask what they can say about *all* graphs. A typical beginning might be a student suggesting that graphs all have points. What can you say about the points of a graph? You can say how many points there are in the graph. This suggests the number of lines, and the number of lines at each point. With some prompting, this leads to discoveries of relations among these concepts. I often have to encourage students to explore this elementary level of concepts, since they tend to move into more complex ideas quickly, omitting basic vocabulary that is needed to state more complicated concepts.

From then on each student works on finding properties concerning graphs. They write up results to hand in after class. I read and make individual comments on these and return them during the week. In class students discuss ideas they have had the previous week: problems they thought up and perhaps solved, hypotheses, examples, counter-examples, proofs. I take notes on ditto stencils describing what goes on in class: definitions, open questions, theorems and the proofs given. The students are thus free to think and participate in the discussion without fear of forgetting what went on during class. The notes are run off and available for students to pick up the following day. I do some rewriting and organizing of the presentations in these notes.

The class meetings are often noisy and charged with excitement. Sometimes several people speak simultaneously. Small groups of students working on a particular problem often talk with one another, temporarily ignoring the rest of the class. Sometimes someone presents a proof to the whole class, while the others attempt to tear it apart. Some proofs stand up under the scrutiny; in others flaws are found. This often results in more work on the part of the students trying to find a valid proof. In every class one or more students discover results which have only appeared in print within the last *40* years, and a few students have proven minor results which are genuinely new.

I do not tell students when they are covering known results, nor when they are venturing off into new areas of Graph Theory. In this way all the material is equally new and potentially solvable for them. I do occasionally help them formulate definitions once the class has decided on the essential content of the definition. I also offer possible nomenclature for properties, although my suggestions are often rejected. Often making up suitable names for properties takes up much time and energy. In some subject areas it could be important for students to know the usual name for a particular concept. Graph Theory, however, is a relatively new area of mathemetics in which nomenclature is not yet standardized. Thus, it does not seem to be a great disadvantage for students to select either one of the usual names for a property or a related, suggestive name.

The students are asked not to read any texts or articles on Graph Theory. For a few of the students who have had difficulty thinking of problems to consider I did provide some papers during the class. On those occasions when the class seems to be out of ideas for new directions, I sometimes mention ideas that had been dropped earlier, or hint at possibilities inherent in things they have already discussed. I try to give minimal help so that the class is oriented towards the concerns and experience of the class members rather than mine. The last two weeks of class I show students some of the books available on Graph Theory, which they could read in the future if they choose. Some students have gotten interested enough in Graph Theory to pursue it at a graduate school level; some do not do any work beyond the class.

I believe this course is an excellent experience in mathematical thinking—in discovering where problems come from and ways of approaching a question that is not at the end of a chapter with sample problems worked out. One student from this class recently wrote me a note in which she said: "Our class time went by entirely too fast. I really don't know how much time I devoted to the course but every second was worth it. I had to think for myself but no one told me what to think or how to think it. This was great. I had freedom to do what I wanted and consequently I was willing to work harder and feel that I learned much more... ."

Graph Theory is particularly well suited to this method of learning since the background needed is slight. In terms of mathematics class, an introduction to any easily axiomatized system (e.g. matroids, or point set topology, or group theory) might be handled in a similar manner. I also believe many other types of classes could use modifications of this discovery method successfully.

It is important for students to do independent thinking; an experience which too few students have prior to the time they try to write a dissertation. It is also valuable for potential teachers to have experience in learning-by-doing—the so-called "discovery approach"—so that they may be more prone to try such methods in their own teaching and be more aware of the problems and advantages inherent in such methods. I have been pleased and surprised by the amount and quality of work students are able to do in this discovery class and would like to see similar teaching methods used in other types of courses.